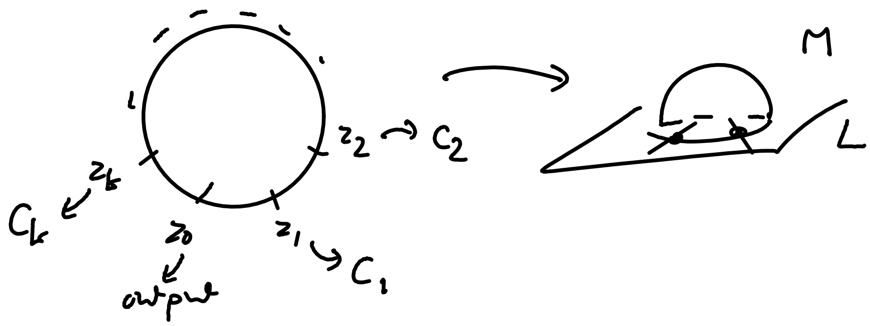
V. Gripp - 17/3/11 - Continuation

Recall: FOOO: $\quad CF_\alpha(L,L) = C_\alpha(L,\Lambda)$

$$m_k(c_1,\dots,c_k) = \sum_{\beta\in\pi_2(M,L)} ev_{0*}\left([\bar{\mathcal{M}}_{0,k+1}(M,L;J,\beta)]\cap ev_1^* c_1 \cap\dots\cap ev_k^* c_k\right) T^{\omega(\beta)}$$



except: • $\beta = 0$ only allowed for $k \geq 2$
• m_1 has extra term = classical boundary.

A_∞-relations: $\displaystyle\sum_{\substack{p+q=k\\ p\geq 1,\, q\geq 0}} \pm m_p(\dots, m_q(\dots), \dots) = 0.$ ($\to$ curved A_∞-structure or A_∞ str. w/ m_0)

Idea: $\partial\bar{\mathcal{M}}_{0,k+1} = \coprod_{p+q=k} \bar{\mathcal{M}}_{0,p+1} \times_{ev} \bar{\mathcal{M}}_{0,q+1}$

$\Rightarrow$ given $c_1 \dots c_k \in CF(L,L)$,

$$(\partial\bar{\mathcal{M}}_{0,k+1})\cap ev_1^* c_1 \cap \dots = \coprod \underbrace{\bar{\mathcal{M}}_{0,p+1}\cap ev_1^* c_1 \cap \dots \times_{ev} \bar{\mathcal{M}}_{0,q+1}\cap \dots}_{m_p(\dots m_q(\dots)\dots)}$$

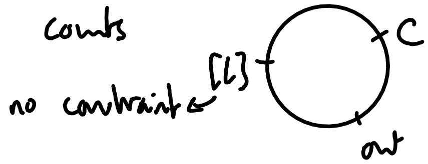
<u>except</u> ∂classical terms in m_1's

$$\| $$

$$\underbrace{\partial(\bar{\mathcal{M}}_{0,k+1}\cap ev_1^* c_1\cap\dots)}_{m_1^{cl}(m_k(\dots))} \pm \underbrace{\bar{\mathcal{M}}_{0,k+1}\cap ev_1^*(\partial c_1)\cap\dots}_{m_k(m_1^{cl}(\cdot),\dots)} \pm \dots, \quad \dots$$

Good case: **Def.** L is weakly unobstructed if m_0 = multiple of $[L]$.

Point: $m_2(c,[L]) = c$ because counts



$\Rightarrow$ extra freedom of moving 2nd pt

$\to$ excess dim. unless disc is constant.

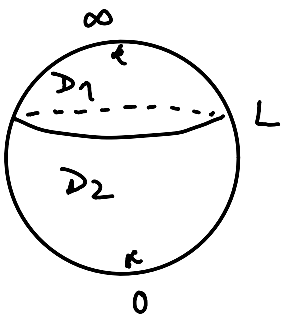
So: $m_1(m_1(C)) = \pm m_2(C, m_0) \mp m_2(m_0, C) = \pm C \mp C = 0$ ✓

<u>Fukaya category of S^2:</u> either one curved A_∞-cat., or more carefully, a <u>collection</u> of honest A_∞-cat's indexed by "charge" param. $\lambda \in \Lambda_{\mathbb{C}}$:

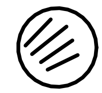
$$\mathrm{Fuk}(S^2, \lambda) = \{\text{weakly unobstr. } (L, \nabla) \text{ s.t. } m_0 = \lambda [L]\}$$

<u>Point</u>: $L \in \mathcal{F}uk(S^2, \lambda)$, $L' \in \mathcal{F}uk(S^2, \lambda')$ $\Rightarrow$ $\partial^2 = \lambda - \lambda'$ on $CF(L, L')$.

Consider $L \subset S^2$, ∇ with holonomy $e^{i\theta}$



observe: $\mu(\beta) = 2.(\beta \cap \{0, \infty\})$

by additivity and $\mu($  $) = 2$.

Since $\dim \mathcal{M}_{\text{disc}} = n - 3 + \mu(\beta) = \mu - 2$,

for m_0 we only care about $\mu = 2$ discs: D_1 and D_2

Let $A = \text{area}(S^2)$, $a = \text{area}(D_1)$

$A - a = \text{area}(D_2)$

$$\Rightarrow m_0 = e^{i\theta} T^a [L] + e^{-i\theta} T^{A-a} [L] = \left(z + \frac{T^A}{z}\right) [L]$$

where $z = e^{i\theta} T^a$

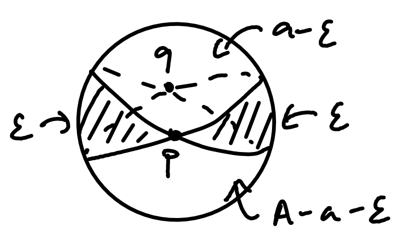
The coeff^t $z + \frac{T^A}{z}$ will play an important role in mirror...

What about $HF(L, L)$?

consider $m_1([p])$: (for dim. reasons, we'll only care about this one)

again counts D_1 & D_2, and $m_1([p]) = e^{i\theta} T^a [L] - e^{-i\theta} T^{A-a} [L]$.

[Why minus sign? <u>eg:</u> consider



$$\partial p = \left(T^{\omega(\varepsilon)} - T^{\omega(\varepsilon)}\right) q$$

$$\partial q = \left(e^{i\theta} T^{a-\varepsilon} - e^{-i\theta} T^{A-a-\varepsilon}\right) p$$]

i.e. $m_1([p]) = \left(z - \frac{T^A}{z}\right)[L] \quad = \left[z\frac{\partial}{\partial z}\left(z + \frac{T^A}{z}\right)\right][L].$

- if $z - \frac{T^A}{z} \neq 0$ then $m_1([p]) =$ mult of $[L] \rightsquigarrow HF(L,L) = 0$
- if $z - \frac{T^A}{z} = 0$ then $HF(L,L) \cong H^*(S^1; \Lambda_{\mathbb{C}})$.
  (as vector spaces)

<u>Q:</u> when is $e^{i\theta} T^a = e^{-i\theta} T^{A-a}$? $\iff \begin{cases} \theta = 0 \text{ or } \pi \\ a = \frac{A}{2} \end{cases}$

i.e. $L =$ <u>equator</u>, $hol(\nabla) = \pm id$.